Oscillations & Waves

1. Simple Harmonic Motion (S.H.M)
2. Superposition of S.H.M.
3. Wave Motion
4. Sound Wave

A 

B 

Result = A + B

$\Rightarrow R = l_1 + l_2$

One Harmonic

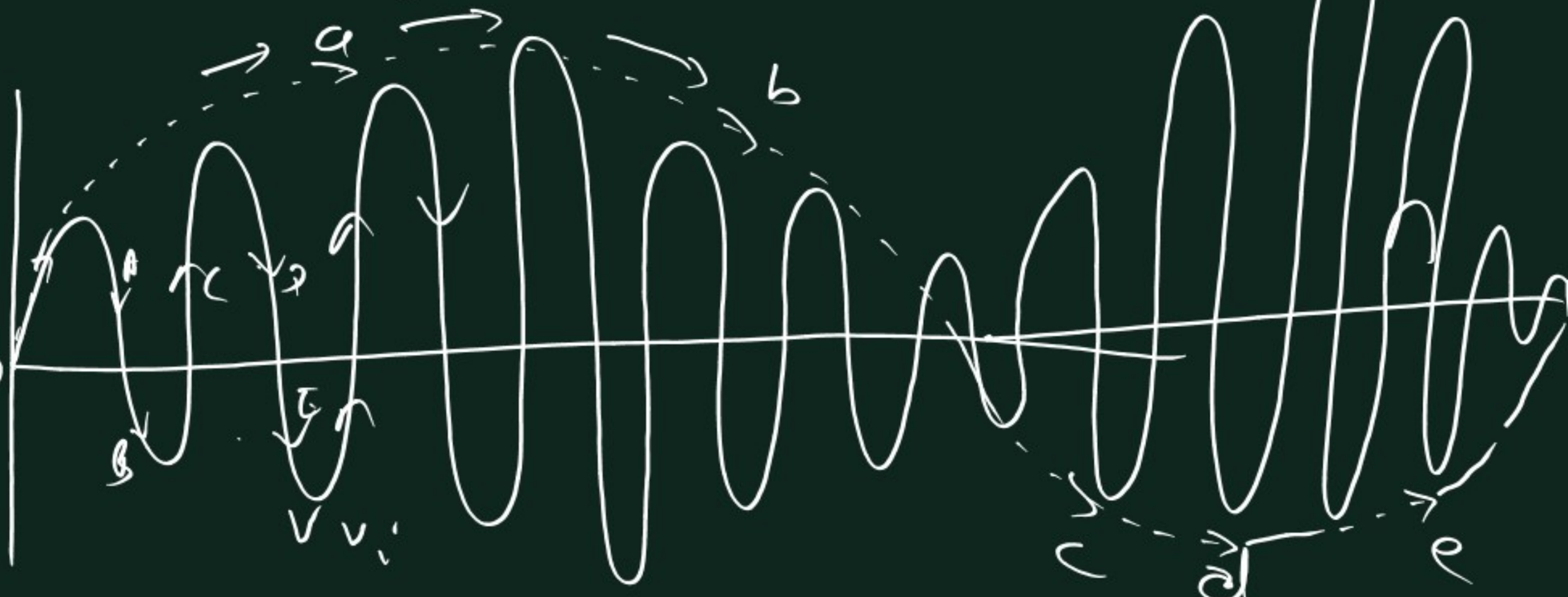
1.

1st Harmonic

2nd Harmonic

3rd Harmonic

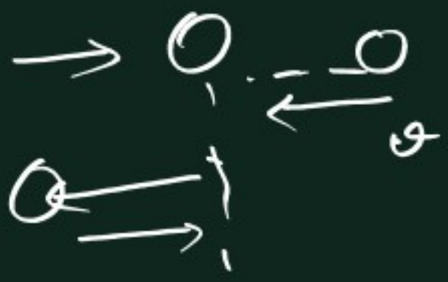
2.

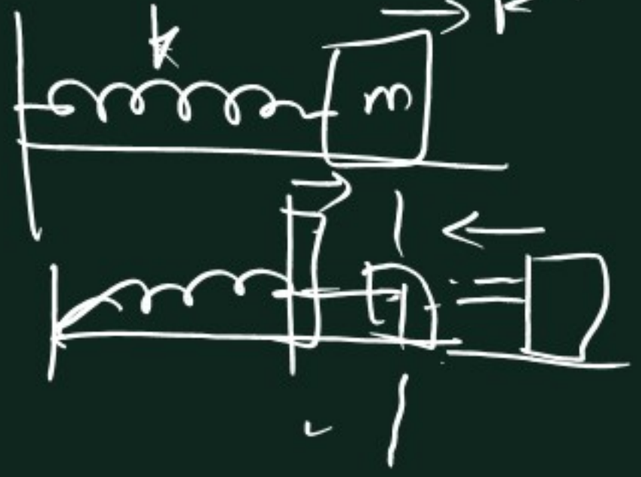


OABCD
→ Type I

Oabcde
→ Type II

S.H.M





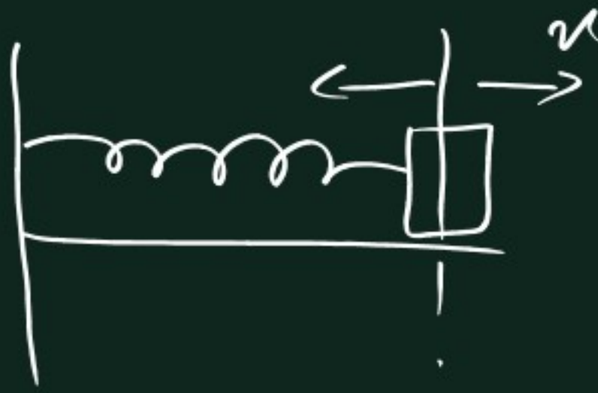
$t=0$

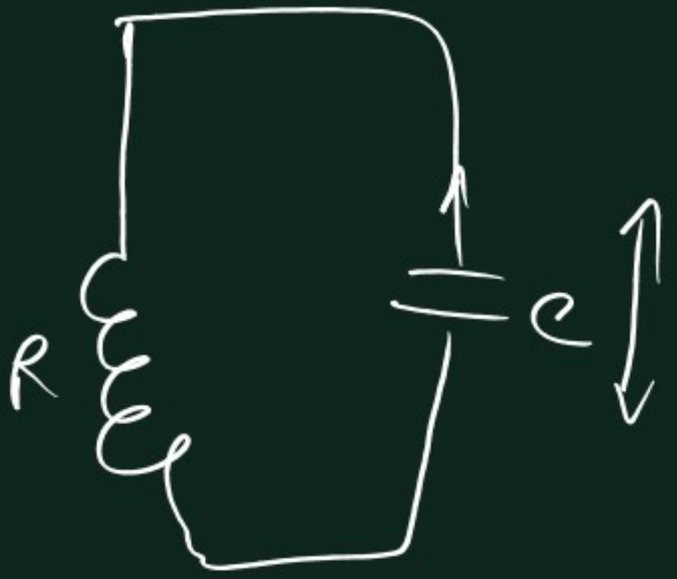
$F_{ext} = F$

$t=t$

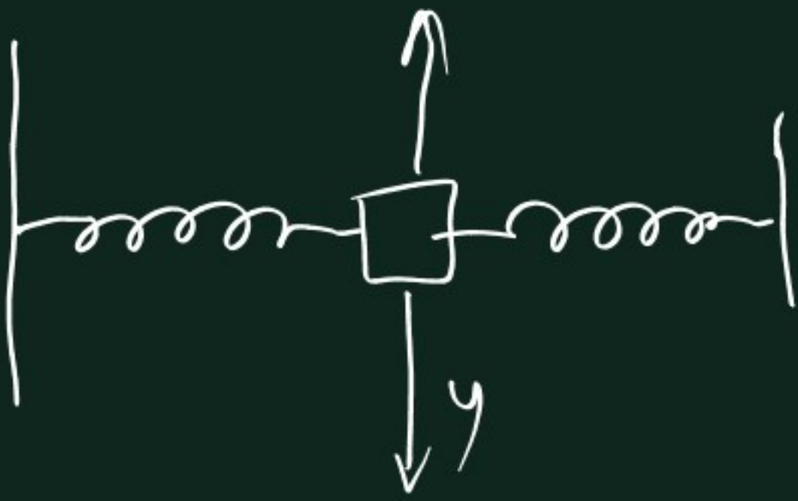
$F_{ext} = 0$

$x \to$ small





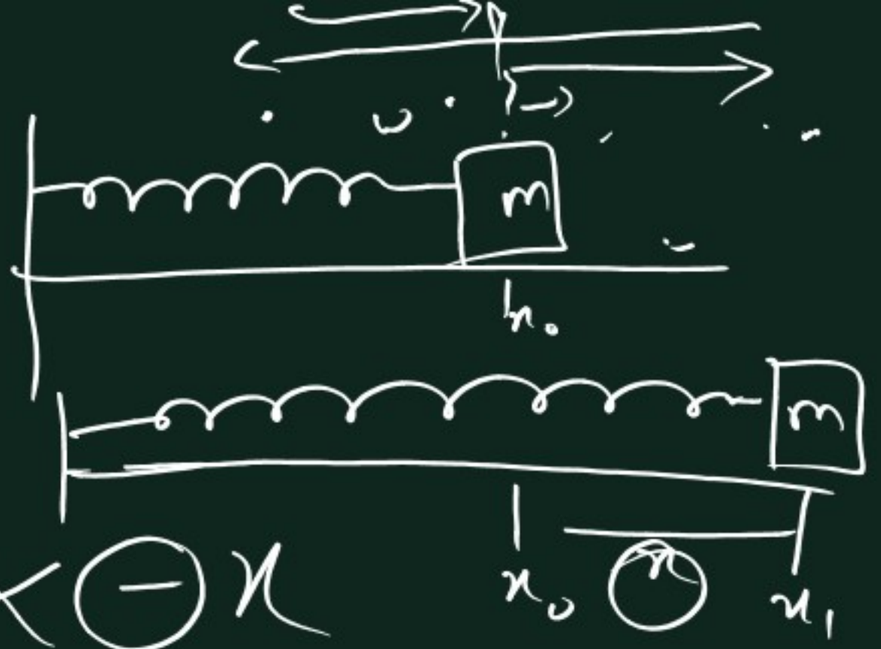
x



① Only one variable

Oscillation is

very small perturbation
around its mean positi



$F_{restoring} \propto (-) x$

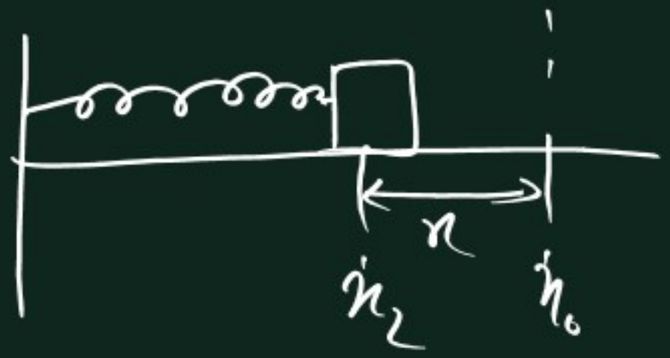
$\omega_0^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

$F_{restoring} = -kx$

$m \frac{d^2x}{dt^2} = -kx$

$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m} x = 0 \Rightarrow$



$\frac{k}{m} = \omega_0^2$

$$\frac{d^2(x)}{dt} + \omega_0^2 x = 0$$

$$D \equiv \frac{d}{dt}$$

$$(D^2 + \omega_0^2)x$$

$$D^2 x + \omega_0^2 x = 0$$

$$(D^2 + \omega_0^2) x = 0$$

$$D^2 + \omega_0^2 = 0$$

$$D^2 = -\omega_0^2$$

$$D = \sqrt{-\omega_0^2} = \pm i\omega_0$$

i = imaginary axis (term)

$i^2 = -1 \Rightarrow \boxed{i = \sqrt{-1}}$

$$e^{\alpha t} = \cos\alpha t + i\sin\alpha t$$

$$e^{-\alpha t} = \cos\alpha t - i\sin\alpha t$$

$$\Rightarrow x = Ae^{i\omega_0 t} + Be^{-i\omega_0 t}$$

$$= A(\cos\omega_0 t + i\sin\omega_0 t) + B(\cos\omega_0 t - i\sin\omega_0 t)$$

$$x = A(\cos\omega_0 t + i\sin\omega_0 t) + B(\cos\omega_0 t - i\sin\omega_0 t)$$

$$= (A+B)\cos\omega_0 t + (Ai - Bi)\sin\omega_0 t$$

$A+B \Rightarrow C$

$Ai - Bi \Rightarrow D$

$$= C\cos(\omega_0 t) + D\sin(\omega_0 t)$$

$5 = 3 + 2$

$\cos^2 t + \cos^2 t = 1$

$a^2 = c^2 + d^2$

$a^2(\sin^2\phi + \cos^2\phi) = c^2 + d^2$

$a^2 = c^2 + d^2$

$C = a\sin\phi$

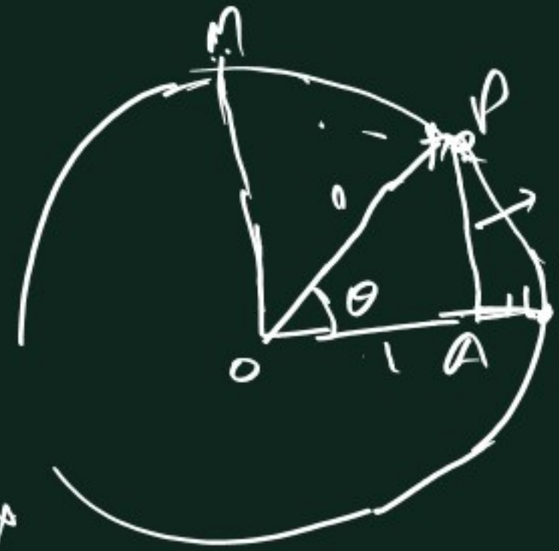
$D = a\cos\phi$

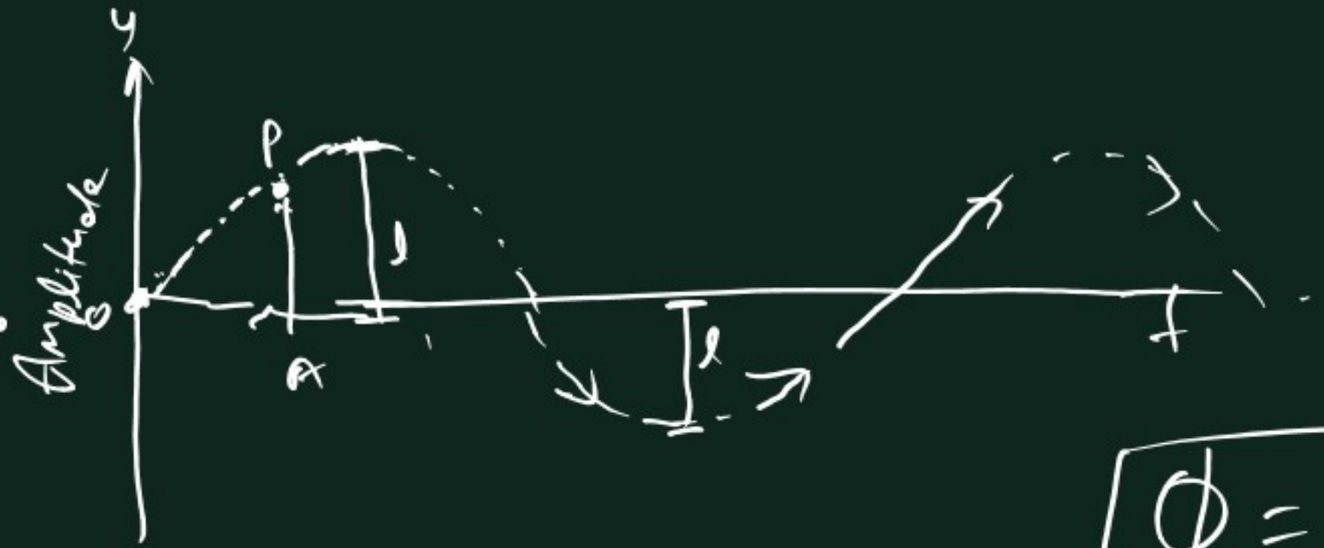
$x = a\sin\phi\cos\omega_0 t + a\cos\phi\sin\omega t$

$= a\sin(\phi + \omega_0 t)$

$x = a\sin(\omega_0 t + \phi)$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$





$$AP = a \sin\theta$$

$a = l$

$\omega t = \theta$

$\phi = 0$

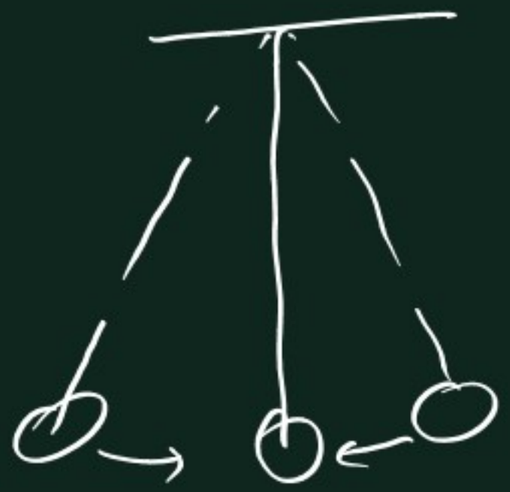
$$y = a \sin(\omega t + \phi)$$

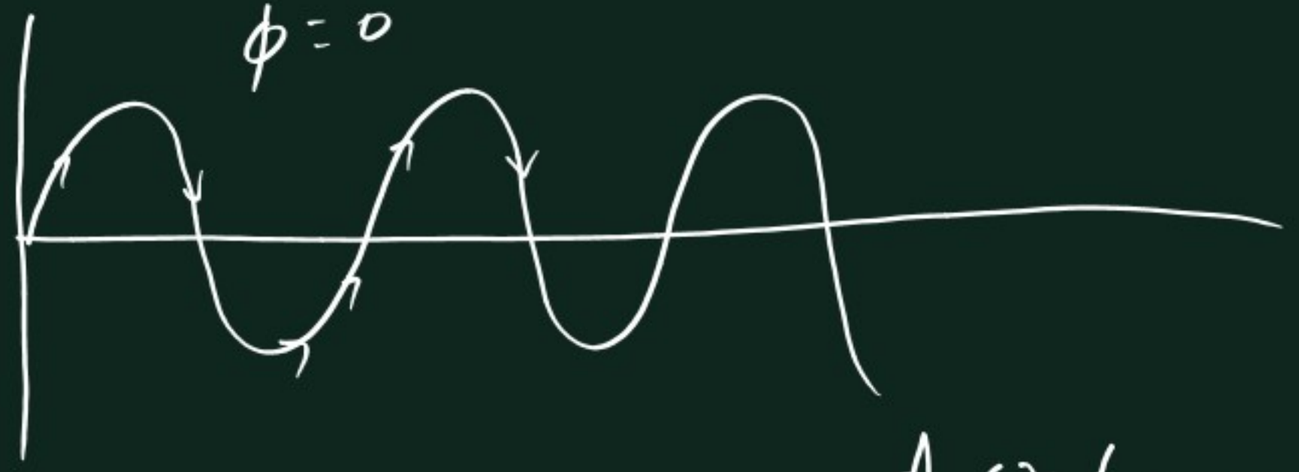
$x =$

$\phi = 0$

$\phi = \frac{\pi}{2}$

$\phi = \theta_1$





$$x = a\sin(\omega t + \phi)$$

$$\text{velocity} = \frac{dx}{dt} = a\omega\cos(\omega t + \phi)$$

$$v = \omega a\left[\sqrt{1 - \sin^2(\omega t + \phi)}\right]$$

$$\frac{d\sin(ax+b)}{dx}$$

$$\frac{d\sin(ax+b)}{d(ax+b)} \times \left[\frac{d(ax+b)}{dx}\right]$$

$$\cos(ax+b) \times a$$

$$a\cos(ax+b)$$

$$v = a\omega \left[\sqrt{1 - \sin^2(\omega t + \phi)}\right]$$

$$= a\omega \sqrt{1 - \left(\frac{x}{a}\right)^2}$$

$$\boxed{v = \omega \sqrt{a^2 - x^2}}$$

$$x = a \sin(\omega t + \phi)$$

$$\sin(\omega t + \phi) = \frac{x}{a}$$

$$v = a\omega \cos(\omega t + \phi)$$

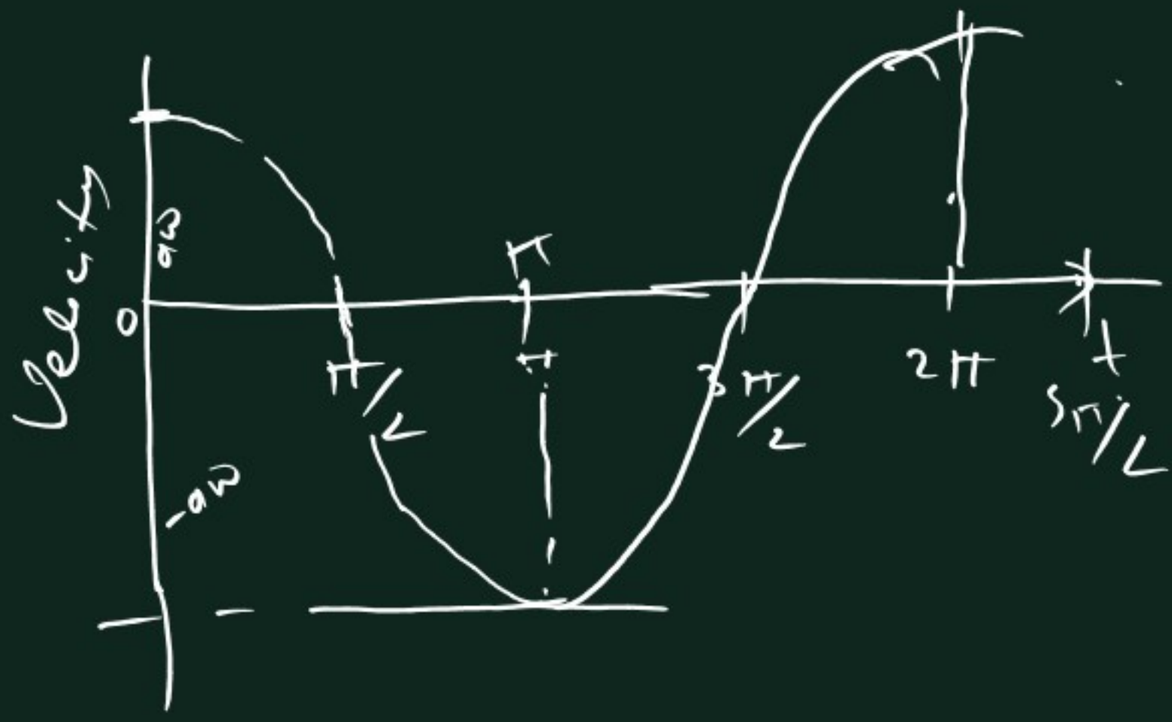
$t = 0$

$\phi = 0$

$v = a\omega$

$\omega t = \frac{\pi}{2}$; $v = \cos\left(\frac{\pi}{2}\right) a\omega = 0$

$\omega t = \pi$

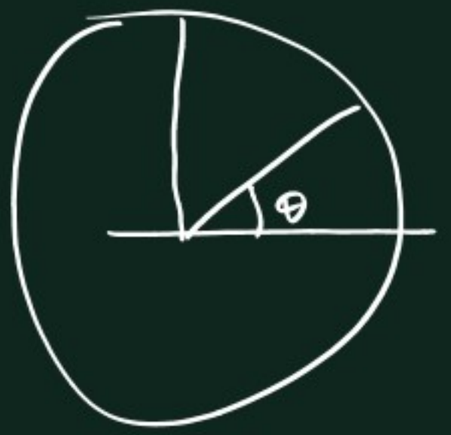


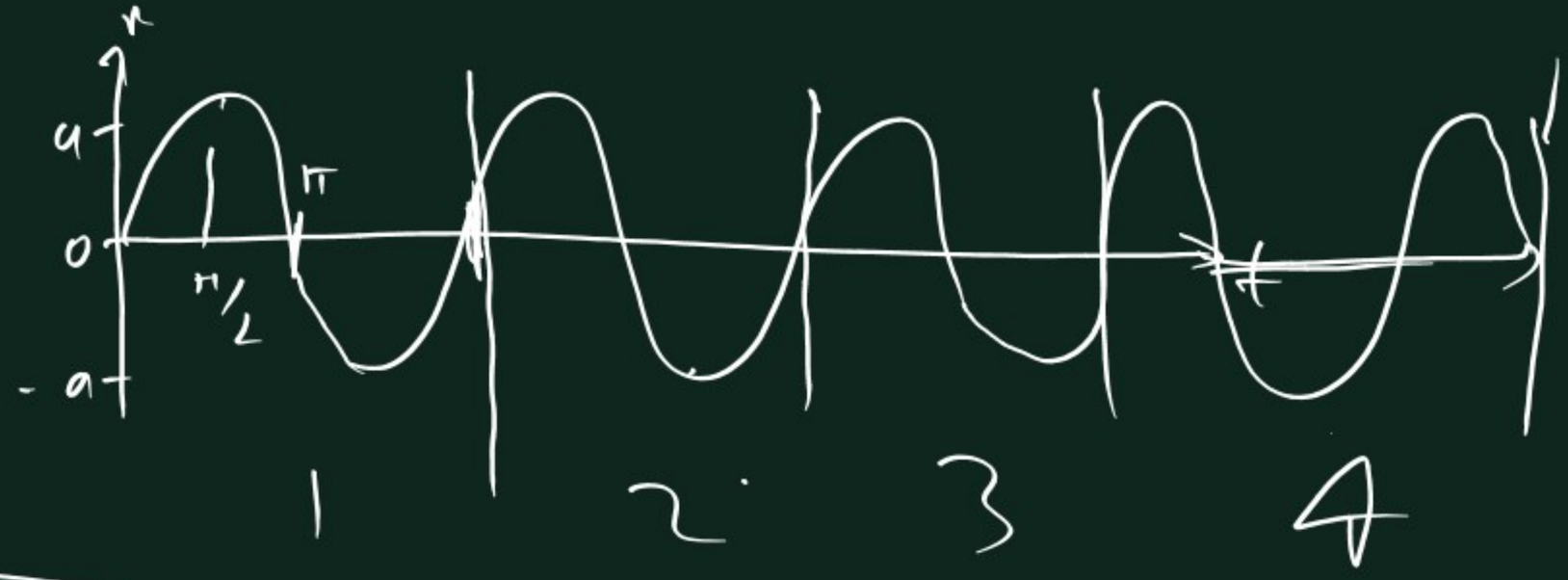
$$v = a\omega \cos(\omega t + \phi)$$

$$\text{acceleration} \Rightarrow \alpha = \frac{dv}{dt} = a\omega^2 \sin(\omega t + \phi)$$

$$\alpha = \omega^2 (a \sin(\omega t + \phi))$$

$$\alpha = \omega^2 x$$





$\theta = \omega t$

$x = a \sin(\omega t)$

Occurrance = 4 ... 5 min

inverse of Time period = $\frac{1}{T}$ (SI Unit)

t at one period = T

$\omega t = \frac{\pi}{2}$

$\omega t = \pi$

$\omega t = \frac{3\pi}{2}$

$\omega t = 2\pi \Rightarrow$ Period $\Rightarrow \boxed{\omega T = 2\pi}$

$\boxed{\omega = \frac{2\pi}{T}}$

$$f = \text{frequency}(f) = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

$$T = 2\,\text{min}$$

$$f = \frac{1}{120\ \text{sec}}$$

$$f = 0.008\ (\text{s}^{-1})$$

$$f = 100$$

$$x = a\sin(\omega t + \phi)$$

$\phi = 0$

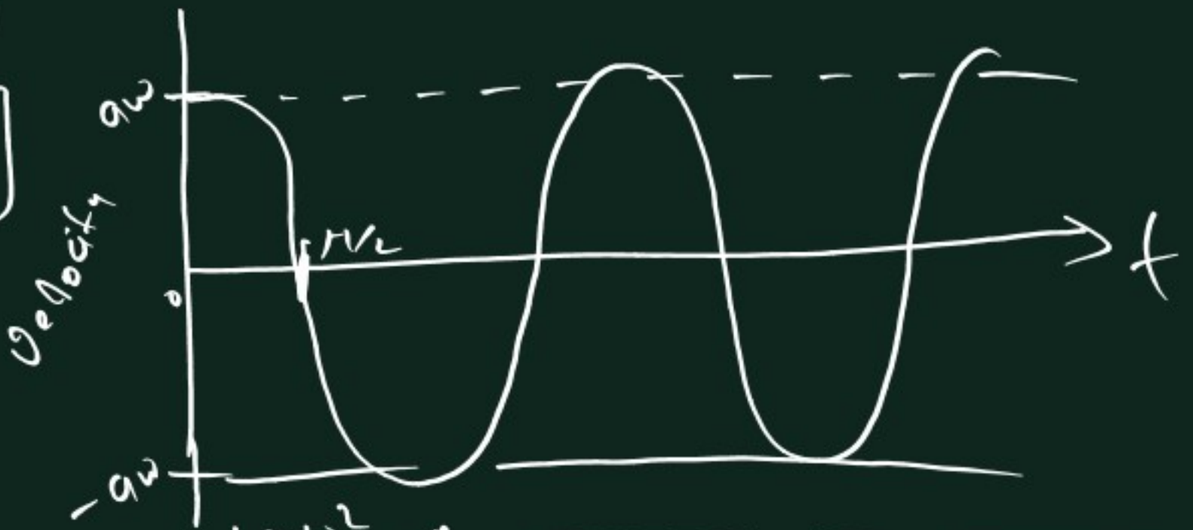
$t = 0$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

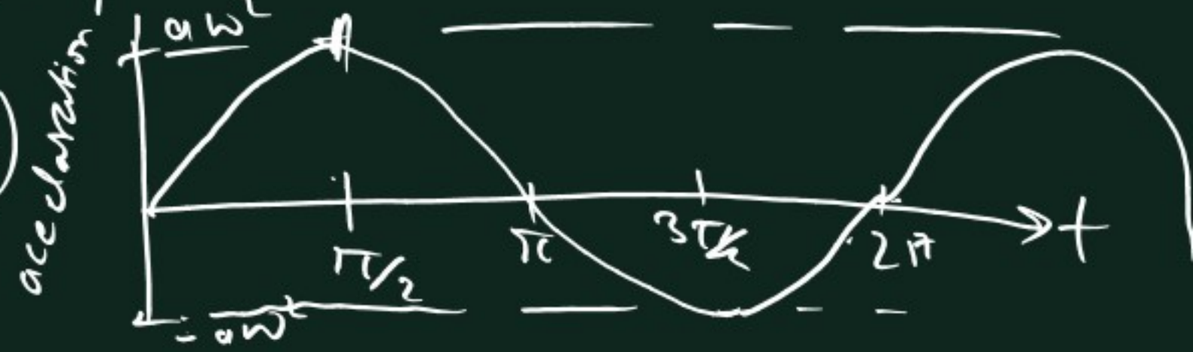
$$v = a\omega\cos(\omega t + \phi)$$

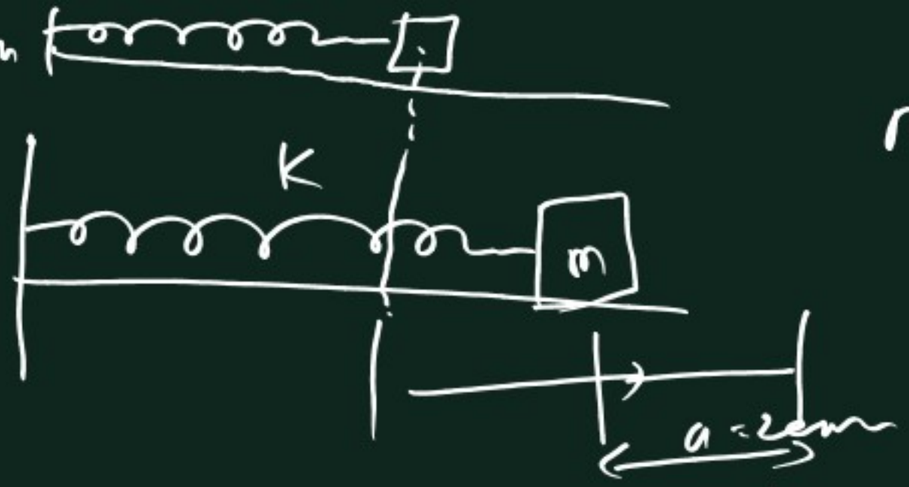
$$= \omega\left[\sqrt{a^2 - x^2}\right]$$



$$\alpha = a\omega^2\sin(\omega t + \phi)$$

$$= -\omega^2 x$$





$m = 10\,kg$

$F_{rest} = -kx$

$K = 5\ N/m$

$6\frac{28}{970}$

$\frac{628}{700} = 8.95$

Time-period $= 8.6 \pm 0.2$ seconds

at $t = 5\ min$

8.4
8.8 $\Rightarrow 8.6 \pm 0.2$

$x = ?$
$v = ?$
$a = ?$

8.4
8.8

$\omega = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{5}{10}} = \sqrt{0.5}$

$= 0.707$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.707} = \frac{3.14 \times 2}{0.707}$

$= 1.89$

$\omega = 0.71$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.71} = 0.71$

$x = a \sin(\omega t + \phi)$

$= (2\,cm) \times \sin(267.54)$

$= 2\,cm \times -0.999$

$= -1.998\,cm$

$a = 2\,cm$

$\omega = 0.707$

$t = 5\,min$

$\omega t = 0.707 \times 5 \times 60$

$= 212.1\ rad$

$= 212.1 \times \frac{180}{\pi}\ deg$ ($\pi \downarrow 3.14$)

$= \frac{12,197.54}{360}$

$12\,197.54 = (33 \times 2\pi + 267.54)$

$= 267.54\ deg$

33×360

$= 11,880$

$$x = 1.998 \text{ cm}$$

$$v = \omega\sqrt{a^2 - x^2} = 0.707 \times \left(\sqrt{2^2 - (1.998)^2}\ (cm^2)\right)$$

$$= 0.707 \times 0.089 \text{ cm/s}$$

$$x^2 = (1.998)^2$$

$$4 - 3.992$$

$$= 0.008$$

$$= 0.089$$

$$v = 0.063 \text{ cm/s}$$

$$= 0.063 \text{ cm/s} \times \frac{1\text{m}}{100\text{ cm}}$$

$$= 6.3 \times 10^{-2} \times 10^{-2} \text{ m/s}$$

$$\boxed{v = 6.3 \times 10^{-4} \text{ m/s}}$$

$$\alpha = \omega^2 n = (0.707)^2 \times 1.998 \text{ cm/s}^2$$

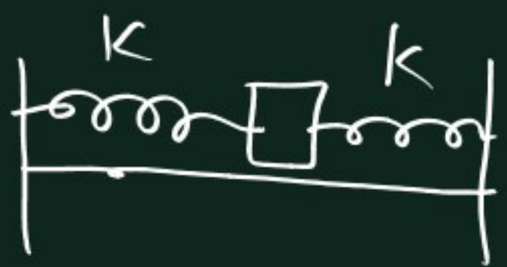
$$= 0.499 \times 1.998 \text{ cm/s}^2$$

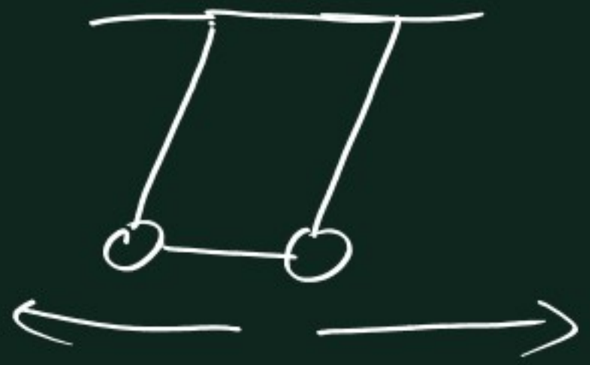
$$\alpha = -0.998 \text{ cm/s}^2$$

$$n = -1.998 \text{ cm}$$

$$v = 6.3 \times 10^{-4} \text{ m/s}$$

$$T = 8.8 \text{ sec}$$





$$\omega^2 = \frac{2\pi}{1 \text{ sec}}^2$$